# Somatorios

# Arithmetic Series

n + (n-1) + (n-2) + … + 2 + 1 = n (n+1)/2

Typical case:

```
For i = 1 to n
    For j = i+1 to n
        do something
    End
End
```

For a fixed value of i, the inner "For" does n-(i+1)+1 = n-i iterations

Iterations of inner "For" when i = 1

Iterations of inner "For" when i = 2

Number of iterations: (n-1) + (n-2) + … + 2 + 1 = (n-1)n/2

# Geometric Series

- 1 + ½ + ¼ + ... + $1/2^i$ + ... = 2
- More generally for 0 ≤ r < 1, 1 + r + r^2 + ... = 1/(1-r)
- A finite Geometric Series can be upper bounded by the infinite one

Ex: 1 + ½ + ¼ + ... + $1/2^n$ ≤ 1 + ½ + ¼ ... = 2

Obs: Usually $1/2^i$ becomes small so quickly that even $\sum_i b_i/2^i$ is at most a constant, for most $b_i$'s we will see

Ex: $\sum_i i/2^i$ is at most a constant

$\sum_i i^2/2^i$ is at most a constant

# General upper bound

- Upper bound each term by the maximum
- $a_1 + \ldots + a_n \leq n \max \{a_i\}$

Ex: Show that $1^2+2^2+\ldots+n^2 = O(n^3)$

- This is the bound we will use the most
- But it is bad in some cases, for instance for Geometric Series
  1 + ½ + ¼ +... + 1/2^n
  - This sum is $\leq$ 2...
  - ... but upper bounding each term gives <= n

# General lower bound

- Want to show $a_1 + a_2 + \cdots + a_n \geq$ value
- Can lower bound each term by minimum

  $a_1 + \ldots + a_n$ >= n min $\{a_i\}$

- But this is usually very bad: look at n + (n-1) + (n-2) + ... + 2 + 1

  = n(n+1)/2, which is $\Omega(n^2)$...

  ...but lower bounding each term gives $\geq$ n*1 = n

**Trick:** Discard smaller terms (usually the smallest n/2)

- Back to example n + (n-1) + (n-2) + ... + 2 + 1

  $\geq$ n + (n-1) + (n-2) + ... + n/2 ← Discarded smallest n/2 terms

  $\geq$ (n/2)*(n/2) = $n^2/4$ ← Lower bounded each term by minimum

# Exercise

Exercise 1: Show that $1^2+2^2+...+n^2 = \theta(n^3)$

# Exercise

Exercise 2: Show that log(n!) is $\Theta$(n log n)

Answer: First we show that log (n!) is O(n log n):

$$\log(n!) = \log n + \log(n-1) + \ldots \log 1 < n. \log n$$

Now we show that log (n!) is $\Omega$(n log n):

$$\begin{aligned} \log(n!) &= \log n + \log(n-1) + \ldots \log 1 \\ &\geq n/2. \log(n/2) \\ &= n/2 (\log n - 1) \end{aligned}$$