

Somatorios

Arithmetic Series

$$n + (n-1) + (n-2) + \dots + 2 + 1 = n(n+1)/2$$

Typical case:

For i = 1 to n

For j = i+1 to n

do something

End

End

For a fixed value of i, the inner "For" does $n-(i+1)+1 = n-i$ iterations

Iterations of inner
"For" when i = 1

Iterations of inner
"For" when i = 2

Number of iterations: $(n-1) + (n-2) + \dots + 2 + 1 = (n-1)n/2$

Geometric Series

- $1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^i} + \dots = 2$
- More generally for $0 \leq r < 1$, $1 + r + r^2 + \dots = \frac{1}{1-r}$
- A finite Geometric Series can be upper bounded by the infinite one

Ex: $1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n} \leq 1 + \frac{1}{2} + \frac{1}{4} \dots = 2$

Obs: Usually $\frac{1}{2^i}$ becomes small so quickly that even $\sum_i b_i/2^i$ is at most a constant, for most b_i 's we will see

Ex: $\sum_i i/2^i$ is at most a constant
 $\sum_i i^2/2^i$ is at most a constant

General upper bound

- Upper bound each term by the maximum
- $a_1 + \dots + a_n \leq n \max \{a_i\}$

Ex: Show that $1^2 + 2^2 + \dots + n^2 = O(n^3)$

- This is the bound we will use the most
- But it is bad in some cases, for instance for Geometric Series
 $1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n}$
 - This sum is ≤ 2 ...
 - ... but upper bounding each term gives $\leq n$

General lower bound

- Want to show $a_1 + a_2 + \dots + a_n \geq \text{value}$
- Can lower bound each term by minimum
$$a_1 + \dots + a_n \geq n \min \{a_i\}$$
- But this is usually very bad: look at $n + (n-1) + (n-2) + \dots + 2 + 1$
 $= n(n+1)/2$, which is $\Omega(n^2)$...
...but lower bounding each term gives $\geq n * 1 = n$

Trick: Discard smaller terms (usually the smallest $n/2$)

- Back to example $n + (n-1) + (n-2) + \dots + 2 + 1$
 $\geq n + (n-1) + (n-2) + \dots + n/2$ ← Discarded smallest $n/2$ terms
 $\geq (n/2) * (n/2) = n^2/4$ ← Lower bounded each term by minimum

Ex: Show that $1^2 + 2^2 + \dots + n^2 = \Omega(n^3)$