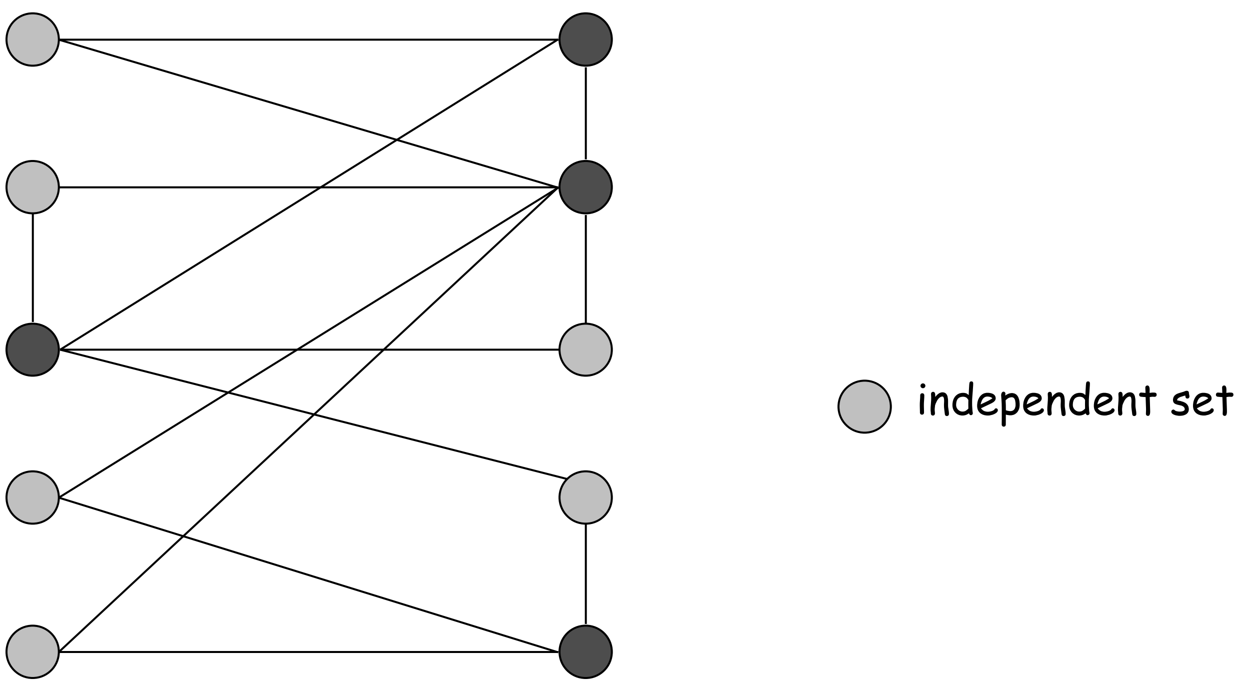
Largest Independent Set

INDEPENDENT SET: Given a graph $G = (V, E)$ what is the largest subset of vertices $S \subseteq V$ such that each edge of E has at most one endpoint in V

Ex. Is there an independent set of size ≥ 6? Yes.
Ex. Is there an independent set of size ≥ 7? No.

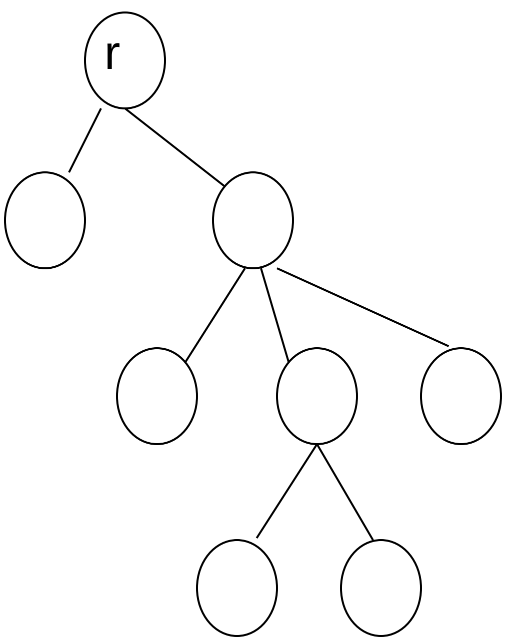


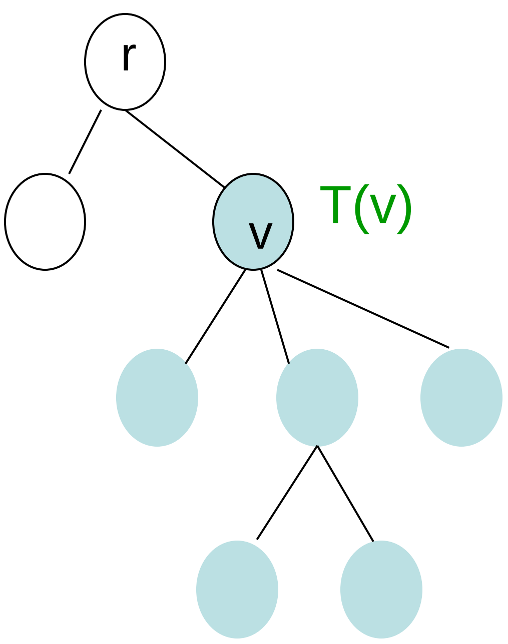
Largest Independent Set

- Intractable problem for general graphs
- Is it efficiently solvable when G is a tree T=(V,E)?

Largest Independent Set

- Select a root r for T
- For a node v in T, let T(v) be the subtree of T rooted at node v





Dynamic Programming Equation

- OPT(v): Largest independent set for T(v)
- We are interested in OPT(r):
  - v is in the optimal solution for T(v)

$$\mathrm{OPT}(v) = 1 + \sum_{\text{w is a grandchild of v}} \mathrm{OPT}(w)$$

  - v is not in the optimal solution for T(v)

$$\mathrm{OPT}(v) = \sum_{\text{w is a child of v}} \mathrm{OPT}(w)$$

Dynamic Programming Equation

- OPT(v): Largest independent set for T(v)
- We are interested in OPT(r):

$$OPT(v) = \begin{cases} 1, \text{ T(v) has one node} \\ \max\left(1 + \sum_{\text{w is a grandchild of v}} \text{OPT(w)}, \sum_{\text{w is a child of v}} \text{OPT(w)} \right) \end{cases}$$

Recursive Algorithm

IndSet(v)
  If v has no children
      M[v]=1
  Else If M[v] is empty
      SomaFilhos←0; SomaNetos←0
      Para todo filho w de v
        SomaFilhos ← SomaFilhos+ IndSet(w)
      Para todo neto w de v
        SomaNetos ← SomaNetos+ IndSet(w)
      M[v]← max(SomaFilhos,1+SomaNetos)
 End if
Return M[v]

Análise

- Cada nó **v** e chamado no máximo duas vezes, uma vez por seu pai e outra por seu avô
- A primeira chamada ao nó **v** tem custo proporcional ao número de filhos mais o número de netos. A segunda chamada custa O(1) .
- Somando o custo de todas as chamadas obtemos O(|V|)

$$\text{custo} = \sum_{\text{v in V}} 1 + |\text{filhos(v)}| + |\text{netos(v)}| \leq 3|V|$$

  – Cada nó contribui com 3 unidades