Analysis of the Algebras of Geographical Operations

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Abstract. This paper addresses the problem of formal definition of the operations on geographical information systems (GIS). Geographical data is divided in two main classes: geo-objects and geo-fields, which represent discrete and continuous representations of reality. We study the operations over geo-fields, geo-objects, and the transformations between geo-fields and geo-objects. This analysis has been used as the basis for LEGAL, a general spatial language, which is used in the SPRING GIS, developed by INPE, with support from IBM Brasil and EMBRAPA.

Keywords. Geographical information systems, mapping, spatial data bases.

1 Introduction

This work discusses the nature of the operations performed on geographical information systems (GIS), based on a formal model of the various types of geographical data. The algebras proposed here are able to perform various classes of spatial analysis, including relatively complex ones.

Since the GIS industry has matured to a point where questions of data structure, algorithms and functionality are becoming standardised, data modelling is seen as playing a critical rôle in determining the usability and adequacy of a system (Goodchild, 1992). This concern has led to a number of conceptual formulations for geographical data models, and to a growing interest in the formal definition of geographical operations.

This paper is part of the conceptual work behind the implementation of SPRING, a geographical information system which integrates the different classes and representations of geographical data. For a description of the first version of SPRING, see Câmara et al. (1992). The algebra described herein is being used as the basis for defining LEGAL, a general purpose query and manipulation language used in the second version of SPRING.

2 Previous Work

Altough the duality between *fields* and *objects* as representations of geographical reality on a GIS is well-established in the literature (Goodchild, 1992), there are very few attempts at providing a unified perspective of geographical operations.

Research on geographical algebra operators has been traditionally divided into two main branches: *manipulation function* on maps and *query and presentation* operations on objects. Tomlin (1990) presents a set of operations on map (most oriented towards the raster representation) called “map algebra”. Egenhofer (1994) discusses the problems of designing a query and presentation language for geographical data (dealing mostly with the vector representation of geographical data).

In an earlier paper (Câmara, Freitas and Cordeiro, 1994b) we outlined the basis of the algebra of an important subclass of geographical data, namely that of *geographical fields* (images, digital terrain models and thematic maps). In this work, we discuss the definition of an algebra of *geographical objects* (another important subclass of geographical data) and the transformations between *geographical fields* and *geographical objects*.

3 A Formal Model for Geographical Data

3.1 Maps and Geographical Elements

Our approach follows the philosophy of data model used in the O_2 object-oriented data base management system (Lécluse, Richard and Vélez, 1991) and is based on the SPRING data model (Câmara et al., 1994a), and considers the following disjoint sets:

- The set **I** of geographical element identifiers.
- The set **C** of geographical class names.
- The universe **U** of *descriptive* attributes $\{\mathbf{A_1},..., \mathbf{A_n}\}$, defined on domains $\mathbf{D(A_1)},...,\mathbf{D(A_n)}$.

Maps

Definition 1. Map.

A set of points M which is a subset of R^2 is called a map. The set 2^M is the power set of M.

Altough this definition is independent of scale and projection considerations, it will be sufficient for our model.

Geo-Fields

A *geographical field* or *geo-field* represents a continuous geographical variable over some region of the Earth.

Definition 2. Geographical Field.

Let M be a map. A geographical field (or geo-field, for short) f is a relation $[id, a_1,...a_n, \lambda]$, where id ε **I**, a_i ε D($\mathbf{A_i}$) and $\lambda: M \rightarrow V$ defines a mapping between points in M and values on a domain V.

The geographical fields can be specialised. The domain *D(V)* can be the entire range of reals, or can limited to descriptive choices. In the first case, geographers normally refer to *digital terrain models*; in the latter, to *thematic maps*. Satellite and aerial images are a special case of DTMs, since a continuous variation (that of reflectance to incident radiation) is usually quantized to a limited range.

Figure 1 shows an image where the M is the region of Manaus and the mapping λ associates to each element of M "its reflectance to the solar radiation on the LANDSAT TM sensor, spectral band 4".

Figure 2 - Landsat image over Manaus

Geo-objects

Geo-objects represent individualizable entities of the geographic realm. They are phenomena that may have one or more *graphical representations*, which correspond to the geo-referenced set of co-ordinates that describe the object's location.

Definition 3. Geographical object

A geographical object (or geo-object) *go* is a relation $[id, a_1,...a_n, geo_1,...,geo_n]$ where id ε **I**, a_i ε $\mathbf{D(A_i)}$ and each geometrical attribute geo_i is such that it assigns a representation of the object *go* in the map M_i. The domain of geo_i is $2^{M}{}_i$. We shall denote the *i-th* descriptive attribute of *go* by $go.\mathbf{A}_i$.

In other words, an object is a unique element that can be represented in one or more points on a map, and which has various descriptive attributes. This definition allows for multiple geometrical representations to be assigned to the same geo-object. Figure 2 shows a geometrical representation of geo-object "Italy", shown in connection of the representations of other countries in Europe.

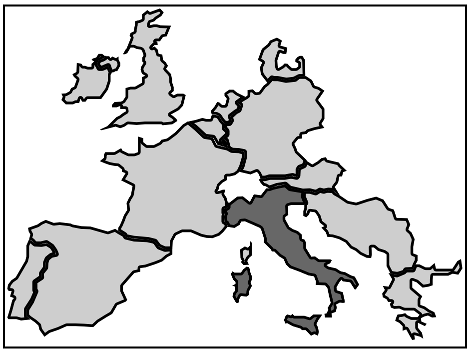


Figure 2 - Example of a geo-object

Geo-object Maps

In a GIS, each geographical object is associated to one or more geographical locations. Since most applications do not deal with isolated elements in space, it is convenient to store the graphical representation of geo-objects together with its neighbours. For example, the parcels of the same city borough are stored and analysed together.

These basic features lead us to introduce the concept of *geo-object maps*, which group together geo-objects for a given cartographic projection and geographical region.

Definition 4. Geo-objects Map.

Let M be a map. A geo-object map *mo* is a relation *[id, M, GO, geo]* such that GO is a set of geo-objects defined by the mapping $geo: GO \rightarrow 2^M$, which assigns, for each geo-object $go \in GO$, a non-empty element of 2^M (a set of locations on the map).

In practice, the mapping *geo(go)* can be interpreted as the *link* between an description of a geographical object and its location on a map M. This situations is typical of large geographical data bases, which include maps in different scales and projections, and over several UTM zones. For example, Figure 3 shows the geo-object "Yang Tse Kiang river" represented in three different maps.

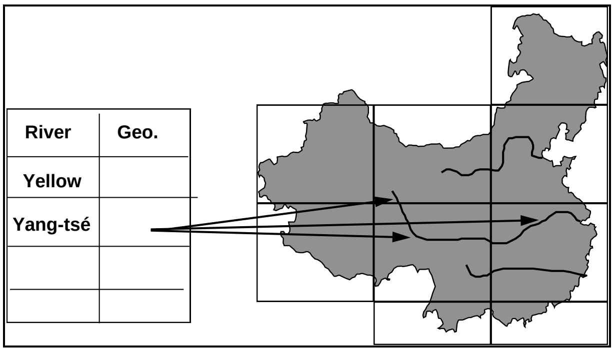


Figure 3 - Multiple representations of a geo-object

3.2 Operations on Geographical Data

There are three main types of geographical algebras:

- *Geo-objects algebra*: selection and query of geo-objects, based on descriptive and spatial properties.
- *Fields algebra:* manipulation of fields.
- *Combined operations*: generation of *geo-object maps* from fields, and generation of *fields* from *geo-objects.*

4 Algebra of Geo-Fields

For the sake of completeness, we shall mention briefly the operations of the algebra of geographical fields, which have been the subject of a previous paper (Câmara, Freitas and Cordeiro, 1994). We shall define four classes of operations: *transformation, point-wise, neighborhood and zonal*.

Definition 5. *Transformation operations on fields.*

The transformation operation τ on a field f_1 defined by λ_1: $M \rightarrow V_1$, generates a field f_2 defined by λ_2: $M \rightarrow V_2$, using a function t such that:

$$\tau(f_1) = f_2 \mid V_2 = t(V_1).$$

In other words, the transformation operation only converts the values of the variable which is represented by the field.

An example would be:

- "reclassify a height map into a thematic map such that: { (0,200 m) $\rightarrow$ "low", (200-500m) $\rightarrow$ "medium", (more than 500 m) $\rightarrow$ "high" }".

Definition 6. Point operations on geo-fields.

The point operation Ξ on a set of geo-fields $f_1, f_2, ..., f_n,$ defined by λ_i: $M \rightarrow V_i$ is such that, given a point function ξ:

$$\Xi(f_1, f_2, ..., f_n) = f_{new} \mid \forall p \in M,$$

$$\lambda_{new}(p) = \xi(\lambda_1(p), ..., \lambda_n(p)).$$

Examples would be:

- "calculate a soil loss equation, given by: (slope map)$^{0.25}$ * (soil ph)^2".
- "calculate a soil aptitude map based on climate, soil, and slope maps, where the conditions are such that a soil is deemed "good for agriculture" if it rains more that 1000 m/year and the soil has a *ph* between 6.5 and 7.5, and the slope is less than 15%".

Definition 7. Neighbourhood operations on geo-fields.

The neighbourhood operation Ψ on a geo-field f_1, defined by λ_1: $M \rightarrow V_1$, is such that, given a function υ:

$$\Psi(f_1) = f_{new} \mid \lambda_{new}(p) = \upsilon(\lambda_1(x), x \in L(p))$$

and the local region $L(p)$ is such that

$$\forall p \in M, \exists L(p) \subset M \wedge p \in L(p).$$

An example of this operation would be:

- "calculate the slope of an elevation map, based on the local derivatives at each point".

Definition 8. Zonal operations on geo-fields.

The zonal operation Z on a numerical geo-field f_1, defined by λ_1: $M \rightarrow V_1$, (where V_1 is the set of reals R), and a thematic geo-field f_2, defined by λ_2: $M \rightarrow V_2$, (where V_2 is a discrete set { $v_1,...v_n$}), and a local function υ is such that:

$$Z(f_1) = f_{new} \mid \lambda_{new}(p) = \upsilon(\lambda_1(x), x \in L(p))$$

and the zonal region $L(p)$ satisfies

$\forall p \in M, \exists L(p) \subset M \wedge p \in L(p)$, such that

$$f_2(x) = v_1 \mid \forall x \in L(p).$$

An example of zonal operations would be:

- "Given an slope map and a soils map, find the average slope for each soil area on the map".

5 Geo-Objects Algebra

5.1 Spatial Relationships

In our model, we shall represent geo-objects as 2D geometries (points, lines and regions). As the operations of the geo-objects algebra may involve spatial restrictions, it is important to define spatial relationships, which may be divided in:

- *topological relationships*, such as "inside" and "adjacent to", which are invariant to rotation, translation and scaling transformations. A formalization of this type of relationships has been proposed by Clementini et al. (1993), based on earlier work by Egenhofer (1989);
- *directional relationships*, such as "above" and "beside". There are many informal proposals (Freeman, 1975), but little formal work for this class of operations;
- *metrical relationships*, derived from the distance operations.

In our work, we shall consider only topological and metrical relationships on R^2, based on the following definitions:

- An *area* A is a 2D set of points of dimension 2, whose interior A^o is connected (with no holes) and which has a connected frontier δA.
- A *line* L is a set of connected points of dimension 1, whose frontier δL is the first and the last point or an empty set in the case of a circular line (an "island"), and its interior L^o is the set of the other points.
- A point P is a set of dimension 0, whose interior P^o is the point itself and whose frontier δP is empty.

To analyse the topological relationships on R^2, Egenhofer (1989) has proposed the use of the *4-intersection matrix*, which represents the relations between the interior and the frontiers of two point sets A and B:

$$\begin{bmatrix} \delta A \cap \delta B & \delta A \cap B^o \\ A^o \cap \delta B & A^o \cap B^o \end{bmatrix}$$

The 4-intersection matrix is not sufficient to uniquely identify all possible situations in the case of relationships between lines and areas and lines and lines. Therefore, Clementini et al. (1993) have proposed to consider the dimension of the intersection between the two sets and have found a minimal set of five relationships (*touch, in, cross, overlap* and *disjoint*) which are applicable to all cases. Câmara (1995, in press) has corrected some inaccuracies on these definitions. The formal definitions of these relationships is given below.

The *touch* relationship is applicable to area-area, line-area, line-line, point-area and point-line situations. A set of points S_1 *touches* another set S_2 when they have points in common, but their interiors do not:

S_1 *touch* $S_2 \Leftrightarrow (S_1 \cap S_2 \neq \emptyset) \wedge (S_1^o \cap S_2^o = \emptyset)$

The *in* relationship is applicable to area-area, line-area, point-area and point-line situations. A set of points is *in* another when their intersection is the first set:

S_1 *in* $S_2 \Leftrightarrow S_1 \cap S_2 = S_1$.

The *cross* relationship is applicable in the case of line-line and line-area situations. A line L *crosses* an area A when their interiors meet and the intersection of the two sets is not the line itself; two lines *cross* when their interiors have a non-empty intersection and this intersections is a set of points of dimension 0:

L *cross* A $\Leftrightarrow (L^o \cap A^o \neq \emptyset) \wedge ((L \cap A) \neq L)$.

L_1 *cross* $L_2 \Leftrightarrow (L_1^o \cap L_2^o \neq \emptyset) \wedge (\dim (L_1 \cap L_2) = 0$.

The *overlap* relationship is applicable to area-area, line-line and point-point situations. Two point sets S_1 and S_2 *overlap* when their intersection is different from them, but forms a set of points of the same dimension:

S_1 *overlap* $S_2 \Leftrightarrow (S_1 \cap S_2 \neq S_1) \wedge (S_1 \cap S_2 \neq S_2) \wedge (\dim(S_1^o \cap S_2^o) = \dim(S_1^o))$.

These situations are illustrated in Figure 4. For a proof of these definitions, please refer to Câmara (1995, in press).

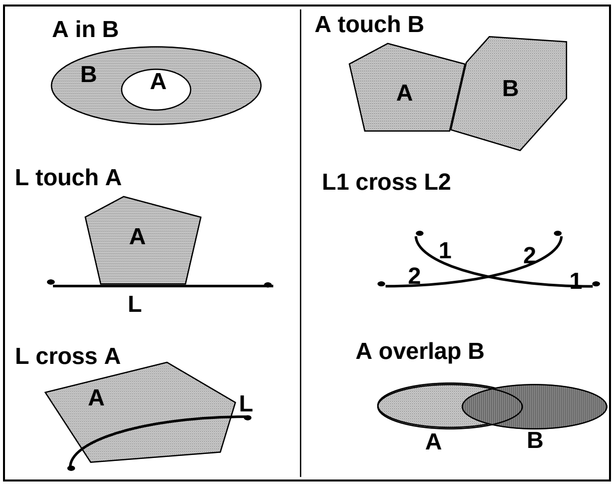


Figure 4 - Examples of topological relationships.

5.2 Operations

We shall restrict ourselves in this paper to the analysis of a subset of the operations on geo-objects, namely to the case when each set of geo-objects *GO* is *homogeneous*, according to the following definition.

Definition 9. Homogeneous set of geo-objects.

A set of geo-objects *GO* is called homogeneous when $\forall\ go_i,\ go_j\ \varepsilon\ GO$, their descriptive attribute domains D(**A**$_1$),..D(**A**$_n$) are the same.

The algebra of geo-objects considered in our work consists of a set of operations with the following restrictions:

- the input is formed by homogeneous sets of geo-objects GO_1, GO_2, ..., GO_n represented respectively in the geo-objects maps mo_1, mo_2, ..., mo_n.
- All geo-object maps mo_1, mo_2, ..., mo_n are defined over the *same* map *M*. This restriction is necessary so that the spatial predicates can be computed.
- The output is a set of objects O_{new} whose geometry is represented in the geo-objects map mo_{new}.

These operations use the topological primitives *touch, in, disjoint, cross* and *overlap,* and metrical relationships, both unary (length, area, perimeter) and binary (distance).

Attribute Selection

Definition 10. Attribute selection

The attribute selection operation σ over a homogeneous set *GO*, given a restriction solely based on the descriptive attributes of a set *GO,* is such that:

$\sigma_{(A_i = a_i)}(GO) = GO' \subset GO,$

$\forall\ go\ \varepsilon\ GO',\ go.\mathbf{A}_i = a_i.$

This is the same operation as the selection on relational algebra, as illustrated by the following example:

- "select all cities in the state of São Paulo with population between 100.000 and 500.000".

Spatial selection

Definition 11. Spatial selection

The spatial selection operation φ over a homogeneous set GO_1 represented in mo_1, given a spatial predicate which relates the geo-objects $go\ \varepsilon\ GO_1$ to a geo-object $go^*\ \varepsilon\ GO_2$ represented in mo_2, where mo_1 and mo_2 are defined over the same map *M*, is such that:

$\varphi_{\ predicate}(GO, go^*) = GO' \subset GO,$

$\forall go \varepsilon GO',\ predicate(geo(go), geo(go^*))$ is true in *M.*

The output of such operation is a subset of the original set, composed of all geo-objects that satisfy the geometrical predicate, as the examples illustrate:

- "select all regions of France which are adjacent to the Midi-Pyrinees regions (which contains the city of Toulouse)".

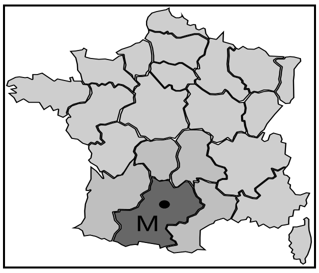


Figure 5 - Example of a spatial selection operation.

Spatial Join

Definition 11. Spatial Join

The spatial join operation θ over two homogeneous sets GO_1 and GO_2 represented in the geo-object maps mo_1 and mo_2 defined over the same map *M*, based on a spatial predicate, is such that:

$\theta_{predicate}\ (GO_1,GO_2)=$

$GO_{new} = \{\ (go_1,\ go_2)\},\ go_1\ \varepsilon\ GO_1,\ go_2\ \varepsilon\ GO_2$

and $predicate(geo_1(go_1),\ geo_2(go_2))$ is true in *M.*

The spatial join is an operation where a comparison between two sets of geo-objects GO_1 and GO_2 takes place, based on a spatial predicate which is computed over the representation of these sets. The name "spatial join" is employed by analogy to the join operation in relational algebra.

The result of the spatial join operation is a *set of object-pairs*, which satisfy the spatial restriction. Examples are:

- "Find all indian reservations located closer than 50 km to the main roads in Amazonia".
- "Find all cities in the state of Ceara which are located close than 10 km from a water reservoir."

In the first example, the answer is a set of pairs of geo-objects (reservation, road) and in the second a set of pairs (cities, reservoir).

6 Transformations between Geo-Fields and Geo-objects

Another set of operations for geographical data concerns the transformations that generate geo-fields from sets of geo-objects (and vice-versa). These transformation operations are of special importance, as the are the link between the two general classes of geographical data.

6.1 Generation of Geo-Objects from Geo-Fields

These operations take as input:

- a set of fields $f_1, f_2, ..., f_n$, defined on a map M, where to each field f_i there corresponds a mapping λ_i: $M \rightarrow V_i$.

The output will be:

- a set *GO* of geo-objects represented in the geo-objects map *mo*, where *mo* is defined over *M.*

We shall consider one important instance of such operations, that of *spatial interpolation*.

Definition 9. Spatial Interpolation

The spatial interpolation operation $\otimes$ over a set of fields $f_1, f_2, ..., f_n$, defined on a map M, where each field f_i is defined by a mapping λ_i: $M \rightarrow V_i$, is such that:

$\otimes\ (f_1, f_2, ..., f_n) = (GO, MO)$,

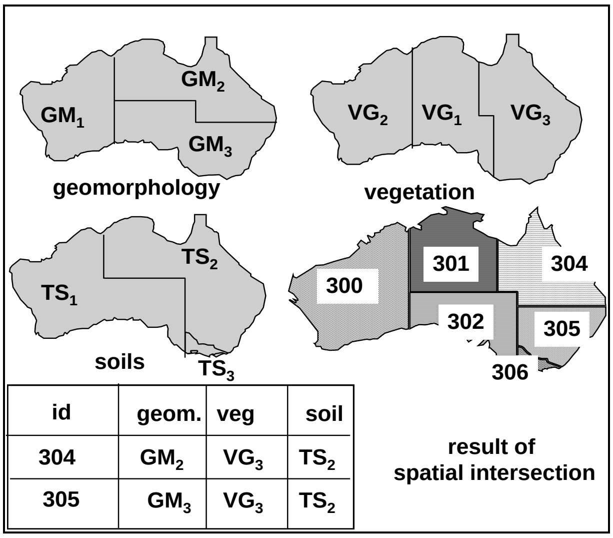
where *mo* is defined over *M*, and

$\forall go\ \varepsilon\ GO,\ go = [id, v_1,...,v_n, geo]$,

where $v_i\ \varepsilon\ D(V_i)$, and given $p\ \varepsilon\ M$,

$p\ \varepsilon\ geo\ (go) \Leftrightarrow\ f_1(p) = v_1 \wedge\ ...\ f_n(p) = v_n$.

This definition corresponds to the generation of a geo-objects maps (typically a cadastral map) from the spatial intersection of a set of geo-fields. This situation occurs, for example, in *zoning* applications, when an overlay of thematic maps is performed to obtain homogeneous zones.



id	geom.	veg	soil
304	GM_2	VG_3	TS_2
305	GM_3	VG_3	TS_2

Figure 6 - Spatial Interpolation Operation

When a cadastral map is created from an overlay of geo-fields, each resulting geo-object inherits all descriptive attributes from the original geo-fields. Consider the following example, as shown in figure 5:

- "Determine the homogeneous regions of Australia, as the intersection of the vegetation, geomorphology and soils maps".

In the GIS literature, the spatial intersection operation is very often wrongly classified as "a special type of spatial join" (cf. Güting, 1994). Altough there are similarities in graphical algorithms used to compute them, the *spatial intersection* operation is conceptually different from the *boolean operations* between geo-fields and from *spatial join* operations between geo-objects.

6.2 Generation of Geo-Fields from Geo-Objects

These operations take as input a set of geo-objects *GO,* represented in the geo-objects map *mo* and generate as output a field f_1, defined on a map M by a mapping λ: $M \rightarrow V$. We shall consider two operations, that of *distance maps* (buffer zones) and that of *attribute reclassification*.

Buffer zones

Definition 10. Buffer zones operation.

Let *go* be a geo-object represented in a geo-object map *mo*, where *mo* is defined over a map *M*. The buffer zones operation Δ is such that, given a distance metric *dist*:

$\Delta(go,\ mo) = f = [id,\ \lambda],\ \lambda: M \rightarrow V$

where $\forall p\ \varepsilon\ M, \lambda(p)= dist(p, geo(go))$.

Figure 7 shows the example of a buffer zone operation.

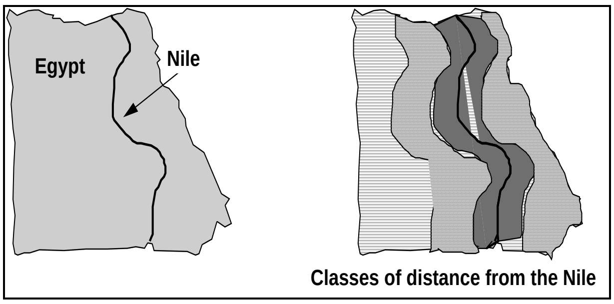


Figure 7 - Example of buffer zones operation

Attribute reclassification

Definition 11. Attribute reclassification operation.

The attribute reclassification operation Ω over a homogeneous set of geo-objects *GO* represented in a geo-objects map *mo* which is defined in a map *M*, is such that:

Ω (*GO*, *mo*, $\mathbf{A}_i$) = f = [*id*, λ], $\lambda: M \rightarrow V$, where $\forall p \; \varepsilon \; M$,

$\lambda(p) = a_i \Leftrightarrow \exists \; go \; \varepsilon \; GO \;|\; p \; \varepsilon \; geo(go)$ and $go.\mathbf{A}_i = a_i$.

From the values of a specific descriptive attribute of a set of geo-objects, a new geo-field is created, whose mapping is defined by the spatial distribution of the chosen attribute. This is illustrated in Figure 8, which shows the operation:

- "For all countries in South America, generate a thematic map with the population growth of each country, divided in classes: { (from 0 to 2% per year), (from 2 to 3%), (more than 3%)}."

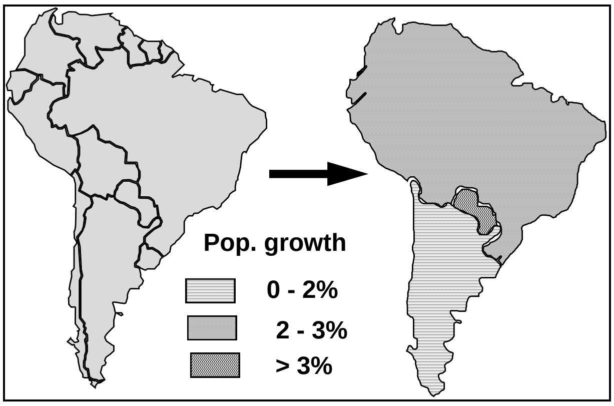


Figure 8 - Attribute reclassification operation

7 The LEGAL language

The analysis of the algebras of GIS operations serves as a basis for the definition of a language for query and manipulation of spatial data, called LEGAL (in Portuguese, "**L**inguagem **E**spaço-**G**eográfica baseada em **Ál**gebra" - Spatial Algebra Language).

The main features of LEGAL are:

- The operations of geo-objects algebra are implemented using extensions of the relational language SQL.
- The fields algebra and the combined field-object operations are implemented by statements which have the same semantic level as the SQL language.

LEGAL é *strongly typed*, and has the following basic types:

- THEMATIC, IMAGE, DTM, which are specialisations of geo-fields;
- OBJECT, for geo-objects;
- OBJECT MAPS, for geo-object maps;
- COLLECTIONS, for storing collections of geo-objects resulting from spatial join operations.

Further work by the authors will concentrate on the definition, implementation and use of LEGAL.

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References

Câmara, G., Freitas, U., Souza, R.C.M., Casanova, M.A. (1992). "SPRING: Processamento de Imagens e Dados Georeferenciados", *Proceedings of V Brazilian Symposium on Computer Graphics and Image Processing (SIBGRAPI 92)*, Águas de Lindoya, 1992, pp. 233-242.

Câmara, G., Freitas, U., Souza, R.C.M., Casanova, M.A., Hemerly, A.S. (1994a). "A General Data Model For Integrating Remote Sensing And GIS Data", in *Proc. ISPRS Commission IV Symposium on Mapping and Geographical Information Systems,* Athens (GA), pages 15-22.

Câmara, G.; Freitas,U.M.; Cordeiro, J.P.C. (1994b) "Towards an Algebra of Geographical Fields". in: VII SIMPÓSIO BRASILEIRO DE COMPUTAÇÃO GRÁFICA E PROCESSAMENTO DE IMAGENS. Anais. Curitiba, 1994, pp. 205-212.

Câmara, G. (1995). "Models, Languages and Architectures for Geographical Information Systems". Ph.D. thesis, INPE, in press.

Clementini, E.; Di Felice, P.; Van Oosterom, P., 1993. "A Small Set of Formal Topological Relationships Suitable for End-User Interaction". in: THIRD INTERNATIONAL SYMPOSIUM ON SPATIAL DATA HANDLING, Proceedings, Singapore, 1993, pp. 277-295.

Egenhofer, M., 1989. *Spatial Query Languages*, Ph.D. Thesis, Depart. of Surveying Engineering, University of Maine, 1989.

Egenhofer, M., 1994. "Spatial SQL: A Query and Presentation Language", *IEEE Transactions on Knowledge and Data Engineering*, vol.6, pp. 86-95, 1994.

Goodchild, M., 1992. "Geographical data modeling". *Computers & Geosciences*, 1992, Vol. 18, no.4, pp. 401-408.

Güting, R.H., 1994. "An Introduction to Spatial Database Systems". *VLDB Journal*, vol. 3, no.4, October 1994.

Lécluse, C.; Richard, P.; Vélez, F. "O_2, an Object-Oriented Data Model". In: Bancilhon, F.; Delobel, C.; Kanellakis,P. (eds.) *Building an Object-Oriented Database System: the Story of O2.* San Mateo, CA, Morgan and Kaufmann Publishers, 1991, cap.4, pp.77-95.

Tomlin, D. (1990) *Geographic information systems and Cartographic Modeling.* Prentice Hall, New York.